Building a Tool for Teaching Mathematical Logic

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Context

Teach a module on an introduction to classical logic. Write an interactive textbook.

- Program Logic (Hoare Calculus)
- Propositional Logic:
  - model, consequence relation, tautology, (un)satisfiability
  - Truthtables
  - Natural deduction calculus
- First-order Logic:
  - model, consequence relation, tautology, (un)satisfiability
  - Natural deduction calculus
  - Equality
- (Mathematical Induction)

Context (Cont'd)

From "Education" to "E-ducation".

- Current situation: Static handouts in form of paper copies and online PostScript/PDF files, weekly lectures and exercise classes to work on worksheets handed out in the previous lecture.
  Prevent collecting and filing. $\leadsto$ see example: gaps.
- Plan: Interactive revision and exercises for all these topics.
  Write interactive course material for a very basic introduction to logic for computer science students
- Starting point: Truthtables.

Example

Semantics of Conjunction

A	B	A ∧ B
T	T	
T	F	
F	T	
F	F	

Note that the four pairs of A and B values in the first two columns exhaust all possibilities (TT, TF, FT, and FF). In the third column, we list the result of the interpretation of A ∧ B according to the truth values of A and B.

In the first line, where A and B have the value T, the result is T again. F else.

Example

Semantics of Conjunction

A	B	A ∧ B
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Example (Cont'd) – Handout

Semantics of Conjunction

A	B	A ∧ B
T	T	
T	F	
F	T	
F	F	

Note that the four pairs of A and B values in the first two columns exhaust all possibilities (TT, TF, FT, and FF). In the third column, we list the result of the interpretation of $\mathrm{A} \wedge \mathrm{B}$ according to the truth values of A and B.
In the first line, where A and B have the value T, the result is T again. F else.

Example (Cont'd) – Online Gaps

Semantics of Conjunction

T
F
F
F

Requirements

- **User-adaptability vs Consistency**
  (above all in nomenclature)
- **User control vs system control**
  (how to balance that the user should be in control of the system but prevent that he is just clicking without any thinking and doesn't learn anything.
- **It should be useful to beginners and advanced students**
  (needs to be easy to use, but not give too much away for beginners, and needs to be not too cumbersome to use for advanced users.)

User-adaptability vs Consistency

How adaptable should a system be? E.g. names:

How to express the logical connectives?

Conjunction:	AND	$\wedge$	&
Disjunction:	OR	$\vee$	|
Implication:	IMPL	$\longrightarrow$	->
Equivalence:	EQV	$\leftrightarrow$	<->

What to offer?

- Choose between those? Yes
- Any others? No
- Allow arbitrary input? No
- Which default? First column

Other decisions about the nomenclature

- How to write placeholders for atoms:
  A, B, C, D, E vs $\phi, \psi, \rho, \sigma, \tau$
  (Leave choice between uppercase Roman vs lowercase Greek)
- How to write truthvalues:
  T, F vs $1, 0$
  (Leave choice between the first and the second.)
- In which order to write truthvalues in a truthtable by default:
  F (or 0) at top vs T (or 1) at top
  (Choice too confusing?)
- Infix vs prefix ...???

Other values

- Colour, e.g. faulty value as red, correct value as green. (What about red-green blind people?)
- Background colour?
  New learning material vs commenting output to user input

What are good defaults?

- Roman vs Greek: Default Roman, since Greek may not be known
- Connectives: Names, since meaning of symbols like $\wedge$ may not be known, browser may not be able to display it.

User control vs system control

- When to offer help?
  When too many failed attempts
- What to do against blind clicking?
  Don't give immediate feedback, explicitly commit.
- What kind of feedback to give?
  General explanation, references back to lecture, fully presented examples
- ...

- Careful balance between useful guidance and patronising, e.g. should the next submission be accepted after a minimal time only?

It should be useful to beginners and advanced students

- Don't expect too much from beginners
  - easy examples, e.g. build up truth tables with just two atoms. (BUT DON'T take examples which are too simple. Trivial cases can be confusing.)
  - don't give away too much, since you cannot expect that the student has understood a concept.
    (e.g. don't tell how many cases are needed in a truth table, how many different atoms do occur)

It should be useful to beginners and advanced students (Cont'd)

- Don't bore people who you assume are advanced by forcing them to fulfil tasks which are trivial for them. Truthtables quickly get boring, e.g. three steps of advancing

Task	Level 1	Level 2	Level 3
knowledge on how many atoms in the formula			
how many cases have to be considered			
structure of truthtable			
which cases are to be considered			
carry over of values of atoms			
compute values for connectives			

It should be useful to beginners and advanced students (Cont'd)

- Don't bore people who you assume are advanced by forcing them to fulfil tasks which are trivial for them. Truthtables quickly get boring, e.g. three steps of advancing

Task	Level 1	Level 2	Level 3
knowledge on how many atoms in the formula	required		
how many cases have to be considered	required		
structure of truthtable	given		
which cases are to be considered	required		
carry over of values of atoms	required		
compute values for connectives	required		

It should be useful to beginners and advanced students (Cont'd)

- Don't bore people who you assume are advanced by forcing them to fulfil tasks which are trivial for them. Truthtables quickly get boring, e.g. three steps of advancing

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how many cases have to be considered	required	given	
structure of truthtable	given	given	
which cases are to be considered	required	given	
carry over of values of atoms	required	required	
compute values for connectives	required	required	

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how many cases have to be considered	required	given	given
structure of truthtable	given	given	given
which cases are to be considered	required	given	given
carry over of values of atoms	required	required	given
compute values for connectives	required	required	required

Demo

What's left to do?

On an educational level

- Randomised generation of new examples
- User input of more examples
- Develop content for the full lecture
- Transform all the LaTeX/PDF slides to the interactive format
- Transform the whole course material to an interactive format

On an MKM level

- Annotate knowledge and make it searchable
- How to make the code robust and adaptable (deal with different logics, e.g., 3-valued logic)
- Build a more sophisticated user-model